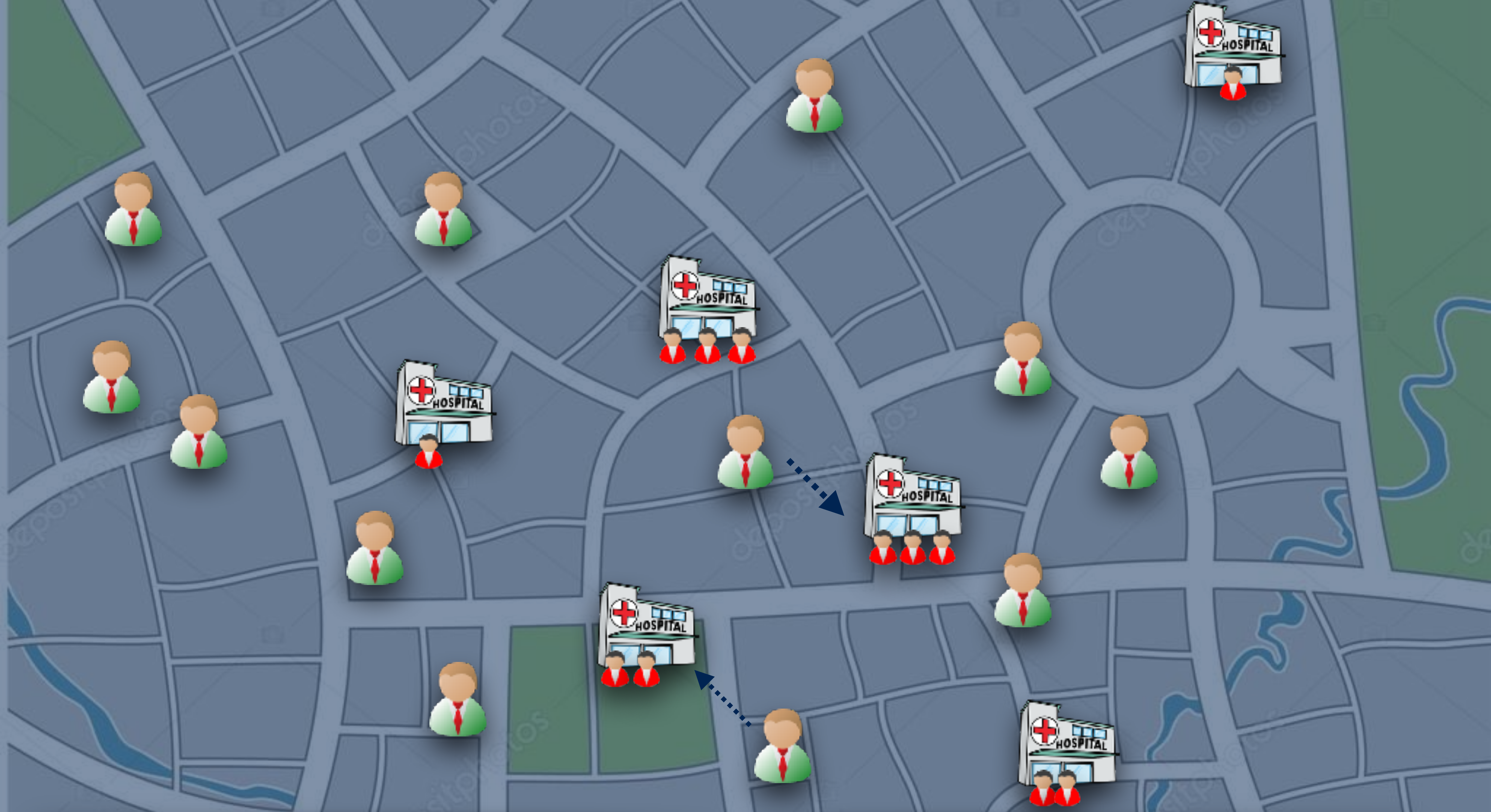
Truthful Facility Assignment with Resource Augmentation: An Exact Analysis of Serial Dictatorship

I. Caragiannis, A. Filos-Ratsikas, S.K.S Frederiksen, K.A. Hansen and Z. Tan

University of Oxford


HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL



Objective: Social cost

Sum of distances between agents and assigned facilities.

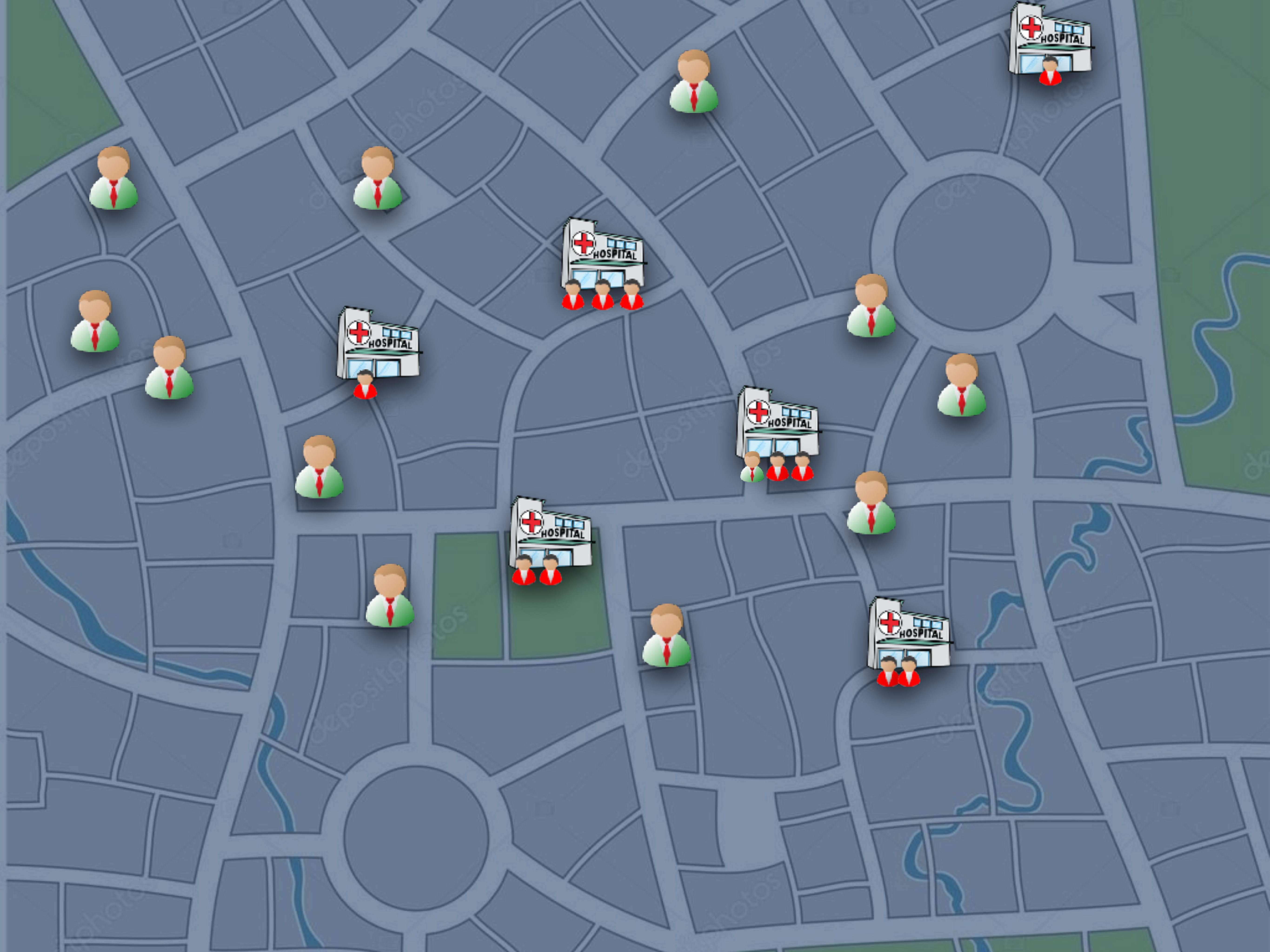
Truthful mechanisms

Incentives to the agents to report their real positions.

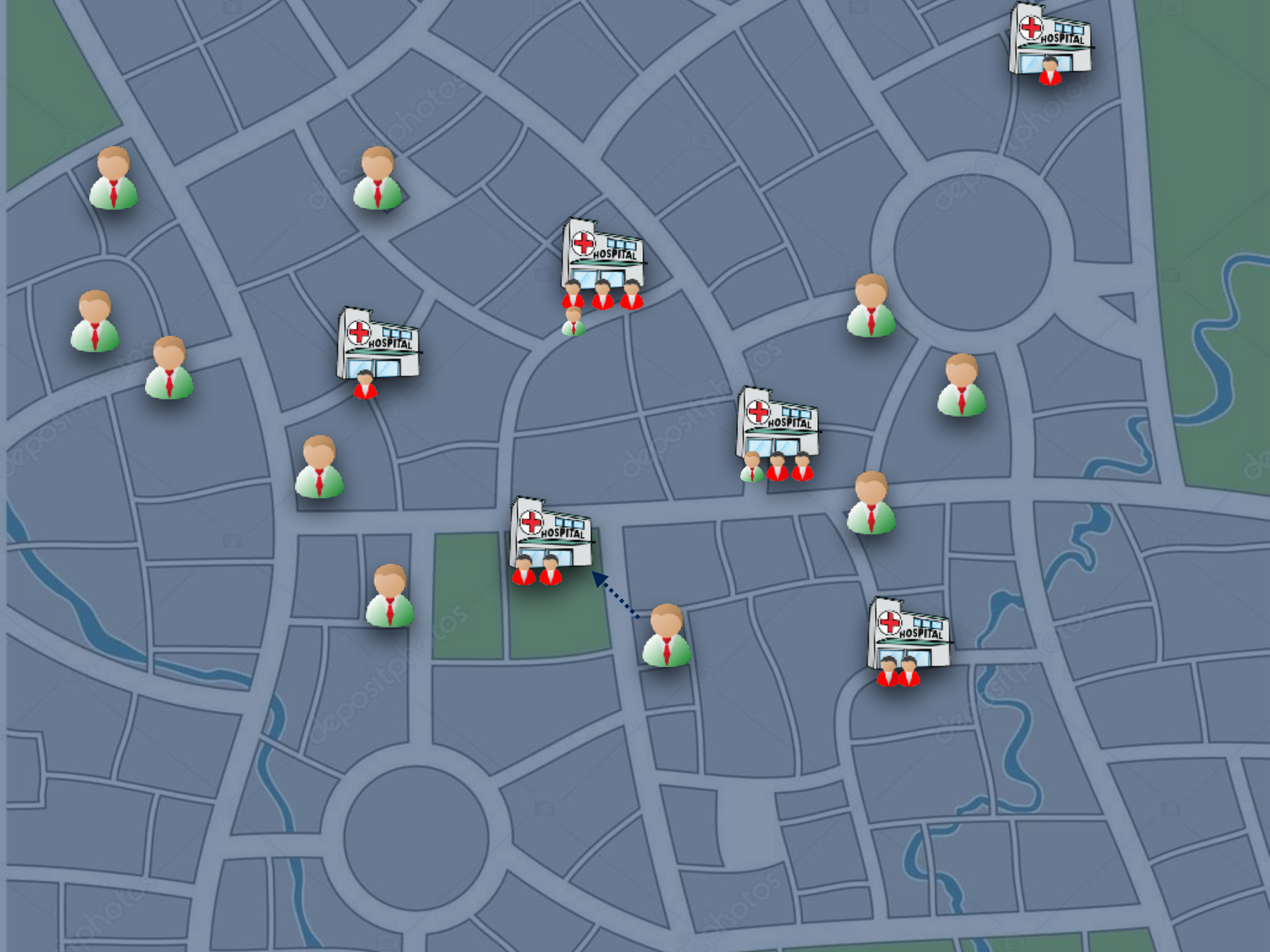
Goal: Design truthful mechanisms with good approximation ratios.

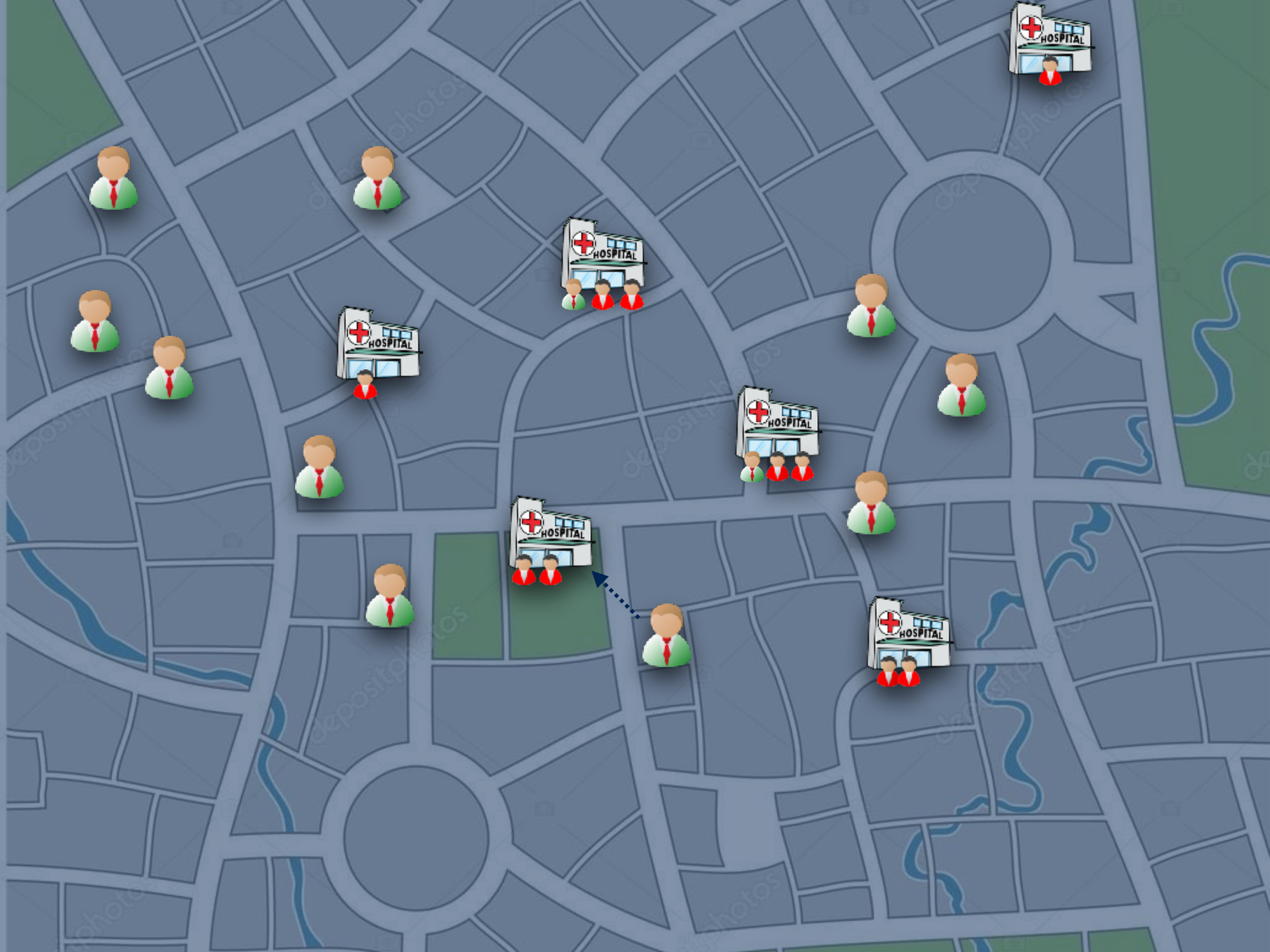

Serial Dictatorship


HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL


HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL


HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL


HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL

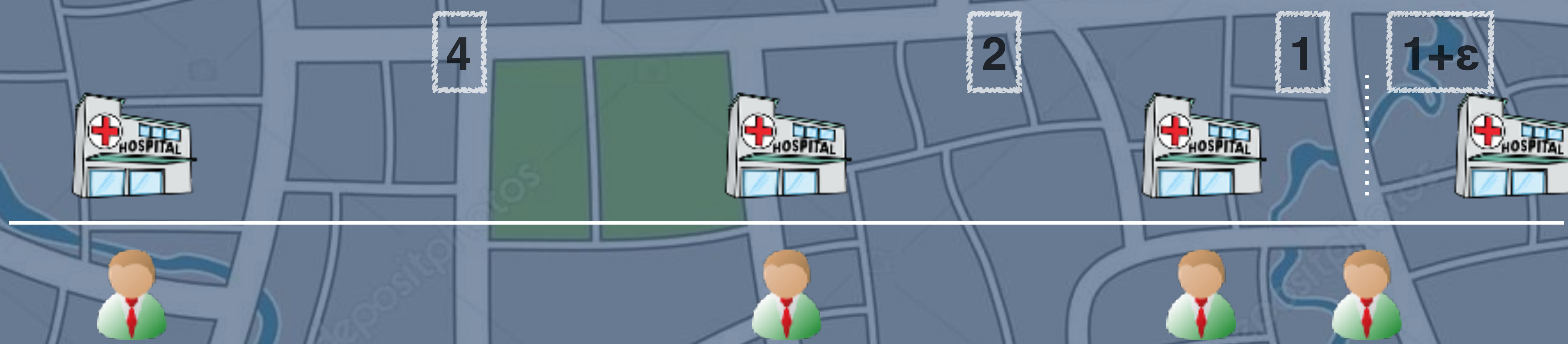

HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL
HOSPITAL

Serial Dictatorship has bad approximation ratio

OPT = $1+\varepsilon$

SD = $2^n - 1$

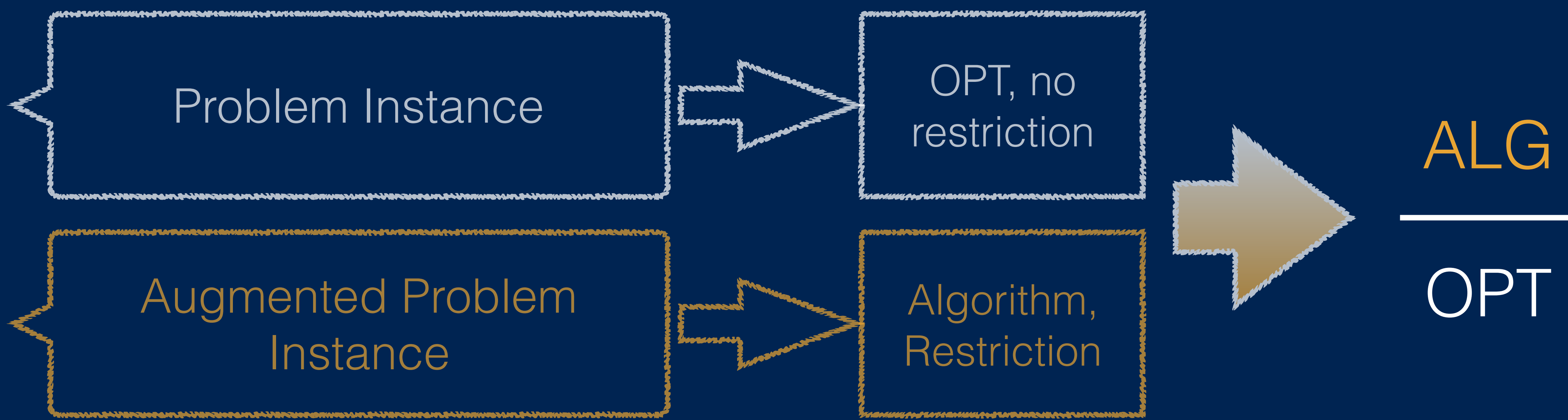
$1 + 2 + 4 + \dots$



Resource Augmentation

- Arguably unfair to compare a mechanism limited by strict constraints like truthfulness to the omnipotent optimal mechanism.
- A reasonable increase in the mechanism's capabilities might have wondrous effects.
- Compare truthful mechanism M with augmented capacities (multiplied by **g**), with the OPT on original capacities.
- **Beyond worst case:** Indication that worst-case instances are rather pathological.

Resource Augmentation



Main Results

Let **ratio**$_{\mathbf{g}}$ be the approximation ratio with augmentation factor **g**. Then it holds that:

ratio(SD) = $\mathbf{2^n -1}$

ratio$_{\mathbf{2}}$**(SD) = log(n+1)**

ratio$_{\mathbf{g}}$**(SD) = g/(g-2)**, when g is at least 3.

ratio(RSD) is between $\mathbf{n^{0.26}}$ and **n**

Proof idea on a single slide

First, consider equivalent formulation where all the capacities are 1.

Proof idea on a single slide

First, consider equivalent formulation where all the capacities are 1.

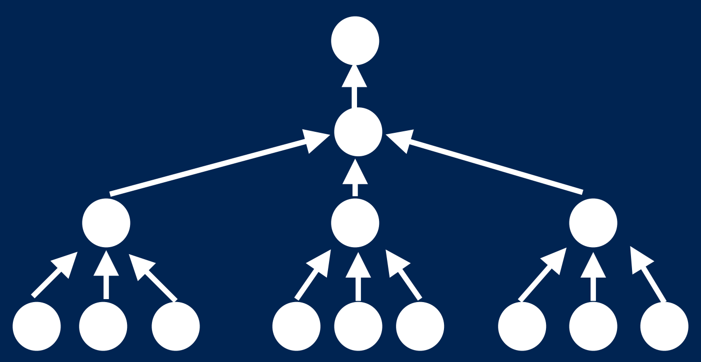
Next, consider graph interpretation
facilities: nodes
agents: edges
edge e=(i,j) -> e goes to i in OPT and to j in SD_g

Proof idea on a single slide

First, consider equivalent formulation where all the capacities are 1.

Next, consider graph interpretation
facilities: nodes
agents: edges
edge e=(i,j) -> e goes to i in OPT and to j in SD_g

Worst case ratio on g-trees

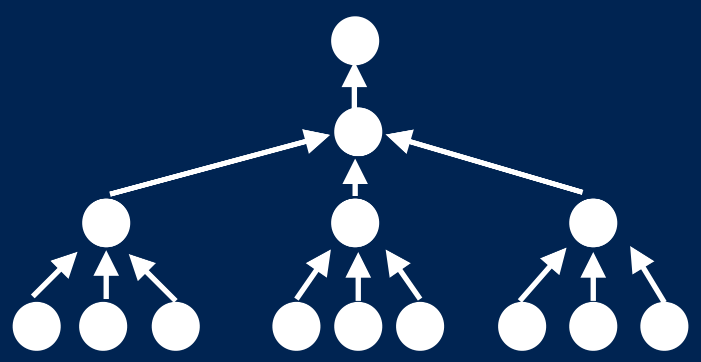


Proof idea on a single slide

First, consider equivalent formulation where all the capacities are 1.

Next, consider graph interpretation
facilities: nodes
agents: edges
edge e=(i,j) -> e goes to i in OPT and to j in SD_g

Worst case ratio on g-trees



Primal

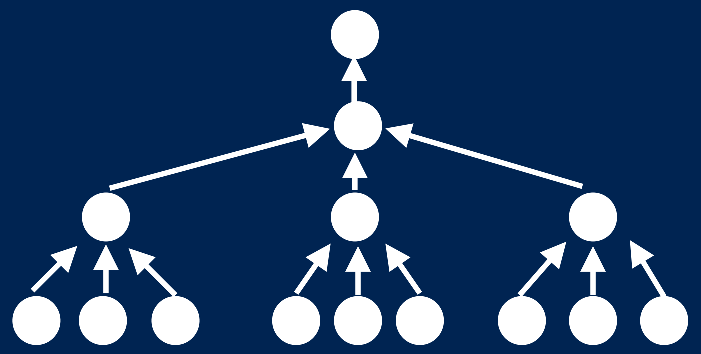
$$
\begin{aligned}
\text{maximize} \quad & \sum_{e \in T} z_e \\
\text{subject to:} \quad & z_e - \sum_{a \in p \setminus \{e\}} z_a \leq \sum_{a \in p} d(A_a, F_{o_a}), e \in T, p \in \tilde{\mathcal{P}}_e \\
& z_e \geq 0, e \in T
\end{aligned}
$$

Proof idea on a single slide

First, consider equivalent formulation where all the capacities are 1.

Next, consider graph interpretation
facilities: nodes
agents: edges
edge e=(i,j) -> e goes to i in OPT and to j in SD_g

Worst case ratio on g-trees



Primal

$$\begin{aligned} \text{maximize} \quad & \sum_{e \in T} z_e \\ \text{subject to:} \quad & z_e - \sum_{a \in p \setminus \{e\}} z_a \leq \sum_{a \in p} d(A_a, F_{o_a}), e \in T, p \in \tilde{\mathcal{P}}_e \\ & z_e \geq 0, e \in T \end{aligned}$$

Dual

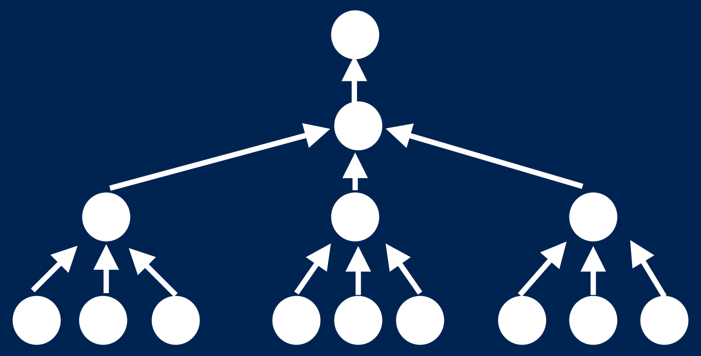
$$\begin{aligned} \text{minimize} \quad & \sum_{p \in \mathcal{P}} x_p \sum_{e \in p} d(A_e, F_{o_e}) \\ \text{subject to:} \quad & \sum_{p \in \mathcal{P}_{e_r}} x_p \geq 1 \\ & \sum_{p \in \tilde{\mathcal{P}}_e} x_p - \sum_{p \in \mathcal{P}_e \setminus \tilde{\mathcal{P}}_e} x_p \geq 1, e \in T \\ & x_p \geq 0, p \in \mathcal{P} \end{aligned}$$

Proof idea on a single slide

First, consider equivalent formulation where all the capacities are 1.

Next, consider graph interpretation
facilities: nodes
agents: edges
edge e=(i,j) -> e goes to i in OPT and to j in SD_g

Worst case ratio on g-trees



Primal

$$\begin{aligned} \text{maximize} \quad & \sum_{e \in T} z_e \\ \text{subject to:} \quad & z_e - \sum_{a \in p \setminus \{e\}} z_a \leq \sum_{a \in p} d(A_a, F_{o_a}), e \in T, p \in \tilde{\mathcal{P}}_e \\ & z_e \geq 0, e \in T \end{aligned}$$

Dual

$$\begin{aligned} \text{minimize} \quad & \sum_{p \in \mathcal{P}} x_p \sum_{e \in p} d(A_e, F_{o_e}) \\ \text{subject to:} \quad & \sum_{p \in \mathcal{P}_{e_r}} x_p \geq 1 \\ & \sum_{p \in \tilde{\mathcal{P}}_e} x_p - \sum_{p \in \mathcal{P}_e \setminus \tilde{\mathcal{P}}_e} x_p \geq 1, e \in T \\ & x_p \geq 0, p \in \mathcal{P} \end{aligned}$$

Find feasible solutions to the dual of low cost

Resource Augmentation

- Online algorithms:
    - ***Weak Adversaries***, e.g. in k-Server.
    - Online Metric Matching.
- Examples present in the Game Theory Community.
    - **Framework:** *Approximate Mechanism Design with Resource Augmentation.*